MA8352-LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT I -VECTOR SPACES

QUESTION BANK

PART A

1. Define vector space
2. Define linearly independent set
3. Define linearly independent set.
4. Define span.
5. Prove that zero vector 0 is unique in $V(F)$.
6. Determine whether $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2X2} / b = c = 0\right\}$ is a vector space under matrix addition and scalar multiplication.
7. Prove that intersection of two subspace is a subspace.
8. Prove that union of two subspace is not a subspace.
9. Determine $(1,1),(3,1)$ whether or not forms a basis of R^2
10. Determind$(1,1,1),(1,-1,5)$ forms a basis of R^3 or not.
11. Relative to the basis $S = \{u_1, u_2\} = \{(1,1),(2,3)\}$ of R^2 find the coordinate of vector V,where $V = (4,-3)$
12. Examine whether or not the vectors $u_1 = (1,1,2), u_2 = (2,3,1), u_3 = (4,5,5)$ in R^3 are linearly dependent.
13. Find the value of k, the vector $V = (1,-2,k)$ in R^3 as a linear combination of vectors $u_1 = (3,0,-2), u_2 = (2,-1,-5)$
14. Find the value of k, the vectors $(2,-1,3),(3,4,-1),(k,2,1)$ becomes linearly independent.
15. Show that the vectors $u_1 = (1,1,1), u_2 = (1,2,3)\ and\ u_3 = (1,5,8)$ span R^3
16. Find the dimensions of W, where W is given by $W = \{(a_1, a_2, a_3, a_4, a_5 \in R^{5/} a_2 = a_3 = a_4\ and\ a_1 + a_5 = 0)\}$
17. Let V_1 be subspace of R^4 given by $V_1 = \{(a,b,c,d)/b - 2c + d = 0\}$. Find basis and dimension of V_1
18. If W_1 and W_2 are subspaces of a vector space $V(F)$. Then their intersection is empty. True or false.
19. Suppose $V = R^2(R)$ and $w = \{(x,y): x^2 = y^2\}$ is a subset of V. Is W a subspace of V?
20. If W is a subspace of the vector space $V(F)$ prove that W must contain 0 vector in V.
21. Is $W = \{(a,0,b)\}: a, b \in R$ a subspace of $R^3(R)$?
22. Is $W = \{(x,y,z): x > 0\}$ a subspace of $R^3(R)$
23. Find the linear span of $S = \{(1,0,0),(2,0,0),(3,0,0) \subset R^3\}$
24. Test whether $S = \{(2,1,0),(1,1,0),(4,2,0)\}$ in R^3 is a basis of R^3over$R$.

25. Find the dimensions of $W_1 = \left\{\begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V: a, b, c \in F\right\}, W_2 = \left\{\begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in V: a, b, c \in F\right\}$
26. What is the dimension of $M_{2X2}(R)$.
27. Let $u_1 = (2,1), u_2 = (-4,-2)$ verify whether $x = (3,5)$ is a linear combination of $u_1$and u_2.

PART B

1. Let V denote the set of ordered pairs of rel numbers. If $(a_1, a_2) and\ (b_1, b_2)$ are elements of V and $c \in R$, we define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$&
$$c(a_1, a_2) = (ca_1, a_2)$$
2. Determine whether the following sets are subspaces of R^3 under the operations of addition and scalar multiplication defined on R^3. Justify your answer.
(a) $W_1 = \{(a_1, a_2, a_3) \in R^3;\ a_1 = 3a_2$ and $a_3 = -a_2\}$
(b) $W_2 = \{(a_1, a_2, a_3) \in R^3;\ 2a_1 - 7a_2 + a_3 = 0\}$
(c) $W_3 = \{(a_1, a_2, a_3) \in R^3;\ a_1 + 2a_2 - 3a_3 = 1\}$
3. For each of the following lists of vectors in R^3, determine whether the first vector can be expressed as a linear combination of the other two.
(a) $(-2,0,3), (1,3,0), (2,4,-1)$
(b) $(3,4,1), (1,-2,1), (-2,-1,1)$
(c) $(5,1,-5), (1,-2,-3), (-2,3,-4)$
4. For each list of polynomials in $P_3(R)$, determine whether the first polynomial can be expressed as a linear combination of the other two.
(a) $x^3 - 3x + 5,\quad x^3 + 2x^2 - x + 1,\ x^3 + 3x^2 - 1$
(b) $-2x^3 - 11x^2 + 3x + 2,\quad x^3 - 2x^2 + 3x - 1,\ 2x^3 + x^2 + 3x - 2$
(c) $x^3 - 8x^2 + 4x,\quad x^3 - 2x^2 + 3x - 1,\ x^3 - 2x + 3$
5. In each part, determine whether the given vector is in the spanof S.
(a) $(2,-1,1),\ S = \{(1,0,2), (-1,1,1)\}$
(b) $(-1,1,1,2),\ S = \{(1,0,1,-1),\ (0,1,1,1)\}$
(c) $-x^3 + 2x^2 + 3x + 3,\quad S = (x^3 + x^2 + x + 1,\ x^2 + x + 1,\ x + 1)$
(d) $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix},\ S = \left\{\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right\}$
6. Show that the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ generate $M_{2\times 2}(F)$
7. Determine x so that the vecors$(1,-1,x-1), (2,x,-4), (0,x+2,-8)$ in R^3 are linearly dependent over R
8. Determine whether the following sets are linearly dependent or linearly independent
(a) $\left\{\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 6 \\ 4 & -8 \end{bmatrix}\right\}$ in $M_{2\times 2}(R)$
(b) $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ in $P_3(R)$
(c) $\{(1,-1,2), (1,-2,1), (1,1,4)\}$ in R^3
(d) $\left\{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ -4 & 4 \end{bmatrix}\right\}$ in $M_{2\times 2}(R)$,Type equation here.

(e) $\{x^4 - x^3 + 5x^2 - 8x + 6,\ -x^4 + x^3 - 5x^2 + 5x - 3, x^4 + 3x^2 - 3x + 5, 2x^4 + 3x^3 + 4x^2 - x + 1,\ x^3 - x + 2\}$in $P_4(R)$.

(f) $(1,2,-3,-4), (3,-1,2,1), (1,-5,8,-7)$

9. Show that if $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $M_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then the span of $\{M_1, M_2, M_3\}$ is the set of all symmetric 2×2 matrices.
10. If $(2,-3,1), (1,4,-2), (-8,12,-4), (1,37,-17), (-3,-3,8)$ generates R^3over R. Find the subset which is a basis of R^3
11. Determine which of the following sets are bases for R^3.
    (a) $\{(1,0,-1), (2,5,1), (0,-4,3)\}$
    (b) $\{(1,2,-1), (1,0,2), (2,1,1)\}$
    (c) $\{(1,-3,-2), (-3,1,3), (-2,-10,-2)\}$
12. Determine which of the following sets are bases for $P_2(R)$.
    (a) $\{-1 - x + 2x^2, 2 + x - 2x^2, 1 - 2x + 4x^2\}$
    (b) $\{1 - 2x - 2x^2, -2 + 3x - x^2, 1 - x + 6x^2\}$
    (c) $\{1 + 2x - x^2, 4 - 2x + x^2, -1 + 18x - 9x^2\}$
13. Do the polynomials $x^3 - 2x^2 + 1, 4x^2 - x + 3$ and $3x - 2$ generate $P_3(R)$?
14. Is $\{(1,4,-6), (1,5,8), (2,1,1), (0,1,0)\}$ a linearly independent subset of R^3? Justify your answer.
15. Show that the vectors $(1,1,2), (1,2,5), (5,3,4)$ do not form a basis of R^3.
16. The set of solutions to the system of linear equations
$$x_1 - 2x_2 + x_3 = 0$$
$$2x_1 - 3x_2 + x_3 = 0$$
is a subspace of R^3. Find a basis for this subspace.
17. The vectors $u_1 = (2,-3,1), u_2 = (1,4,-2), u_3 = (-8,12,-4), u_4 = (1,37,-17), u_5 = (-3,-5,8)$generate R^3. Find a subset of the set $\{u_1, u_2, u_3, u_4, u_5\}$ that is a basis for R^3.
18. Determine the basis and dimension of the solutions of linear homogeneous system $x + y - z = 0;\ -2x - y + 2z = 0;\ -x + z = 0$
19. Find the dimension of the subspace spanned by the following vectors.
    (a) $(2,0,1), (-1,0,1), (1,0,2)$ in $R^3(R)$
    (b) $(1,0,-1), (1,8,14), (0,-4,3)$ in $R^3(R)$
20. The vectors $\{(2,-3,5), (8,-12,20), (1,0,-2,), (0,2,-1), (7,2,0)\}$generates R^3. find a subset of the given set that is a basis for R^3.
21. Let $V = M_{2\times2}(F), w_1 = \left\{\begin{bmatrix} a & b \\ c & a \end{bmatrix} \in V: a, b \in F\right\} w_2 = \left\{\begin{bmatrix} 0 & a \\ -a & b \end{bmatrix} \in V: a, b \in F\right\}$ Find the dimensions of $w_1, w_2, w_1 + w_2$ and $w_1 \cap w_2$.

UNIT II LINEAR TRANSFORMATION AND DIAGONALISATION

PART A

1. Define linear transformation
2. State Rank nullity theorem.
3. If T is linear then $T(0) = 0$
4. Define kernel
5. Define image.
6. Let V and W be the vector spaces and let $T: V \to W$ be linear. Prove that T is one-to-one if and only if $N(T) = \{0\}$
7. Let $T: R^2 \to R^2$ such that $T(1,1) = (1,0,2),\ T(2,3) = (1,-1,4)$ find $T(8,11)$

8. If $F: R^2 \to R^2$ be the linear map defined by $F(1,2) = (2,3)$ and $F(0,1) = (1,4)$, find a formula for $F(a, b)$
9. Check whether $T: R^2 \to R^2$ defined by $T(x, y) = (x + 3,2y)$ is linear or not.
10. Verify whether the following map $T: R^2 = R^2$ defined by $T(x, y) = (y, x)$ is linear
11. If $T: R^3 \to R^2$ is defined by $T(x, y, z) = (x - y, 2z)$ is a linear transformation. Find $N(T)$and $R(T)$. $T(x, y, z) = (x + 2y + z, -x + 5y + z)$. Then rank of T equals
12. Let $T: R^2 \to R^2$ be a linear transformation defined by $T(a, b) = (a + 2, b + 3)$ then find the matrix of T in the basis $\{(e_1, e_2, e_3)\}$ where $e_1 = (1,0), e_2 = (0,1)$
13. Define $T: M_{2\times2}(R) \to P_2(R)$by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a + b) + 2dx + bx^2$.
Let $S = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ and $\gamma = \{1, x, x^2\}$ compute the matrix T
14. Define $T: P_4 \to P_4$by $T\big(p(x)\big) = p'''(x)$. Find the matrix of T relative to the standard basis for P_4
15. Define $T: P_2 \to P_2$by $T\big(p(x)\big) = p(x) + xp'(x)$. Find the matrix of T relative to the standard basis for P_2
16. Find the algebraic multiplicity of all eigen values of $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$
17. Let $T: R^2 \to R^2$ be the linear operator defined by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 2y \\ x - y \end{bmatrix}$. Find the matrix of T
18. Let $T: R^2 \to R^2$ be the linear operator defined by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 3y \\ -x + y \end{bmatrix}$. Find the matrix of T
19. Find the linear transformation $T: V_3 \to V_3$ determined the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ write the standard basis (e_1, e_2, e_3).
20. Show that $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not diagonalizable.

PART B

1. State and prove Rank-nullity theorem.
2. Prove $T(0) = 0$
3. Let V and W be the vector spaces and $T: V \to W$ be linear. Then prove that $N(T)$ and $R(T)$ are subspaces of V and W repectively.
4. Let V and W be vector spaces and let $T: V \to W$ be linear. Then T is $1 - 1$ if and only if $N(T) = \{0\}$
5. Let V and W be a vector spaces of finite dimension and let $T: V \to W$ be linear the the following are equivalent
(a) T is 1-1
(b) T is onto
(c) $rank(T) = \dim V$
6. If $T: R^2 \to R^3$ is defined by $T(x, y) = (x - y, 2x + y)$. Let $B = \{e_1, e_2\}$,$e_1 = (1,0), e_2 = (0,1)$ be the basis for R^2 and $B' = \{(1,1,0), (0,1,1)(2,2,3)\}$ be a basis for R^3. Find the matrix of T in these bases.
7. Let $T: R^2 \to R^3$ be defined by $T(x, y) = (x - y, x, x + 2y)$.Let $B = \{e_1, e_2\}$, $e_1 = (1,2), e_2 = (2,3)$ be the basis for R^2 and $B' = \{v_1, v_2, v_3\}$, where $v_1 = (1,1,0), v_2 = (0,1,1) and\ v_3 = (2,2,3)$ then find the matrix of T

8. Let $T: R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (x + y, y + z)$. find a basis, dimension of each of the range and null apace T and verify dimension theorem. Also check whether T is isomorphism or not.
9. Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (x + 2y - z, , y + z, x + y - 2z)$. find a basis and dimension of null space $N(T)$ and range space $R(T)$. also verify rank nullity theorem.
10. Consider a linear transformation $T: R^3 \to R^3$ with $T(1,1,1) = (111)$, $T(1,2,3) = (-1, -2, -3)$and $T(1,1,2) = (2 ,2, 4)$. Then $T(2,3,6)$ equals.Check whether$T$ is one to one or not.
11. Let $T: R^2 \to R^3$ be defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$. Prove that $T$is linear. Find bases for $N(T)$and $R(T)$. Compute the nullity and rank of T. Determine whether T is one-to-one or onto.
12. Let $T: P_2(R) \to P_3(R)$ be defined by $T\big(f(x)\big) = 2f'(x) + \int_0^x 3f(t)dt$. Find bases for $N(T)$and $R(T)$. Compute the nullity and rank of$T$. Determine whether T is one-to-one or onto.
13. Let $T: P_2(R) \to P_3(R)$ be defined by $T\big(f(x)\big) = xf(x) + f'(x)$. Find the basis of kernel and range of T.
14. Let $T: R^2 \to R^3$ be defined by $T(x, y) = (2x - y, 3x + 4y, x)$. Find $N(T)$and $R(T)$, is T is one-one-one, onto? Justify your answer.
15. Verify rank nullity theore for $T: R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Verify it is $1 - 1$ and onto.
16. Suppose that $T: R^2 \to R^2$ is linear. $T(1,0) = (1,4)$and$T(1,1) = (2,5)$. What is $T(2,3)$? Is T one to one? Is T onto?
17. Let $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2, x_2 - x_3, x_1 + 2x_3)$. Find the bases and dimension of its range and its null space. Also verify dimension theorem.Tone to one? Is T onto?
18. Let $V = P_2(R)$ and T is defined by $T[f(x)] = f(0) + f(1)(x + x^2)$. Test for diagonalisability .
19. Let T be a linear operator on $P_2(R)$ given by $T(f(x)) = xf'(x) + f(2)x + f(3)$. Find the matrix of T and check whether $[T]_B$ is diagonalizable where B is a standard basis.
20. Let T be a linear operator on $P_2(R)$ defined by $T[f(x)] = f(1) + f'(0)x + [f'(0) + f'(1)]x^2$. Test for diagonalisability.
21. For $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} \in M_{2\times 2}(R)$ find an expression for A^n, where n is an arbitrary positive integer.
HINT:To solve this problem we have to find eigen values and eigen vectors$A^n = QD^nQ^{-1}$ where Q= a matrix which is a column of eigen vectors &$Q^{-1} = \frac{adj\ Q}{\det of\ Q}$

22. For each of the following linear operator T on a vector space V, Test for diagonalizability and if diagonalizable, find a basis for V such that the matrix is diagonal matrix
(a) $\boldsymbol{V = P_3(R)}$ **and** $\boldsymbol{T\big(f(x)\big) = f'(x) + f''(x)}$
(b) $\boldsymbol{V = P_2(R) and\ T(ax^2 + bx + c) = cx^2 + bx + a}$

UNIT III INNER PRODUCT SPACE

PART A

1. Define inner product space.
2. Write the axioms of inner product space.
3. Define Frobenius inner product.
4. Define norm.
5. Let $u = (1,3,5), v = (4,5,5)$ in R^3 find the inner product of $\langle u, v\rangle$
6. Show that $\langle (a,b),(c,d)\rangle = ac - bd$ on R^2 is not an inner product on given vector space.
7. Evaluate $\langle 3u - 2v, w\rangle$ when $u = (1,3,-4,2), v = (4,-2,2,1)$ and $w = (5,-1,-2,6)$
8. Check whether $\{u_1, u_2, u_3\}$ is an orthogonal set where $u_1 = \begin{bmatrix}3\\1\\1\end{bmatrix}, u_2 = \begin{bmatrix}-1\\2\\1\end{bmatrix}, u_3 = \begin{bmatrix}1/2\\2\\-7/2\end{bmatrix}$
9. Given $x = (1,2,3)$ and $y = (1,0,1)$ find distance between x and y through inner product.
10. Given $u = (2,3,5), v = (1,-4,3)$ in R^3 then find the angle θ between $u$and v.
11. Let $f(t) = 3t - 5$ and $g(t) = t^2$ in the polynomial space $p(t)$ with inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$. Find the angle θ beween $f$and g.
12. Let $f(x) = 1$ and $g(x) = x$ be functions in the vector space $C[0,1]$ with the inner product defined as $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$ verify that $|\langle f, g\rangle| \le \|f\|\|g\|$
13. Consider the following polynomials in $p(t)$ with the inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$, $f(t) = t + 2$ and $g(t) = 3t - 2$
    (a) Find $\langle f, g\rangle$
    (b) Find $\|f\|$and $\|g\|$
    (c) Normalize f and g
14. Find k so that $u = (1, 2, k, 3)$ and $v = (3,\ k,\ 7, -5)$ in R^4 are orthogonal.
15. Given $x = (1,2,3)$ and $y = (1,0,1)$ find distance between x and y through inner product.
16. Find the norm of $(3,4,0)$ in R^3 with the standard inner product.
17. Consider the vectors $u = (1,5)$ and $v = (3,4)$in R^2. Find $\|v\|$ using the inner product in R^2
18. Verify Cauchy-Schwarz inequality for $u = (1,-1,3)$ and $v = (2,0,-1)$.

PART B

1. State and prove Cauchy-Schwarz inequality and Triangle inequality.
2. State and prove triangle inequality.
3. State and prove Gram Schmidt orthgonalization theorem.
4. Let V be an inner product space and let T and U be a linear operator on V then
    (a) $(T + U)^* = T^* + U^*$
    (b) $(CT)^* = \bar{C}T^*$
    (c) $(TU)^* = U^*T^*$
    (d) $T^{**} = T$
    (e) $I^{**} = I$

5. Let$V = C^3$ where C is the set of complex numbers defined by $\langle x, y\rangle = a_1 R^2 \overline{b_1} + a_2\overline{b_2} + a_3\overline{b_3}$ where $x = (a_1, a_2, a_3)$, $y = (b_1, b_2, b_3)$. verify the inner product space.
6. Show thar$\langle f(x), g(x)\rangle = \int_0^1 f'(t)g(t)dt$ on $P(R)$ is not an inner product on given vector space.
7. Show that the inner product space H with $\langle .,. \rangle$ defined on the interval $[0,2\pi]$ with the inner product $\langle f, g\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(t)\overline{g(t)}dt$ is an inner product space.
8. Let $V = C^3$ with inner product $\langle x, y\rangle = x_1\overline{y_1} + x_2\overline{y_2} + x_3\overline{y_3}$where $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$. Let $x = (2,1 + i, i), y = (2 - i, 2,1 + 2i)$ compute (a) $\langle x, y\rangle$ (b) $\|x\|$, $\|y\|$ and (c) $\|x + y\|$ (d) verify Cauchy's inequality and triangle inequality
9. Use the Frobenius inner product to compute $\|A\|, \|B\|, \langle A, B\rangle$ and $cos\ \theta$ for $A = \begin{bmatrix} 1 & 2+i \\ 3 & i \end{bmatrix}$ and $B = \begin{bmatrix} 1+i & 0 \\ 0 & -i \end{bmatrix}$
10. Find $\cos\theta$ where θ is the angle between
    (a) $u = (1,3,-5,4)$ and $v = (2,-3,4,1)$ in R^4
    (b) $A = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
11. For each of the following inner product spaces V and linear operator T on V, evaluate T^* at the given vector in V.
    (a) $V = R^2, T(a, b) = (2a + b, a - 3b), x = (3,5)$
    (b) $V = C^2, T(z_1, z_2) = (2z_1 + iz_2\ ,(1 - i)z_1)\ ,\ x = (3 - i, 1 + 2i))$
12. Let $V = M_{2\times2}(R), S = \left\{\begin{bmatrix} 3 & 5 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 9 \\ 5 & -1 \end{bmatrix}, \begin{bmatrix} 7 & -17 \\ 2 & -6 \end{bmatrix}\right\}$ and $A = \begin{bmatrix} -1 & 27 \\ -4 & 8 \end{bmatrix}$
    Apply Gram Schmidt process to find orthogonal basis and normalize these these vectors to obtain orthonormal basis. Also compute Fourier co-efficient of the given vector.
13. Apply the gram Schmidt process to the given subset$S = \{(1, -2, -1\ ,3), (3, 6, 3, -1), (1, 4, 2, 8)\}$ and $x = (-1,2,1,1)$ of the inner product space $V = R^4$
    $(a)$to obtain orthogonal basis (b) Normalize these vectors in this basis to obtain an orthnormal basis. (c)compute the Fourier coefficients of the given vector
14. $S = \{(2, -1, -2\ , 4), (-2,\ 1, -5,\ 5), (-1,\ 3,\ 7,\ 11)\}$ and $x = (-11,\ 8, -4,\ 18)$ of the inner product space $V = R^4$
15. Apply the Gram Schmidt process to the given subset $S = \{(1, i, 0), (1 - i, 2, 4i)\}$ and $x = (3 + i, 4i, -4)$ of the inner product space $V = R^3$
16. Using gram Schmidt orthogonalization process construct an orthogonal set from the given set $S = \{(1,0,1), (0,1,1), (1,3,3)\}$of R^3. Also find the Fourier coefficient of the vector $(1,1,2)$ with respect to the resultant orthogonal vectors.
17. Apply the Gram-Schmidt process to the vector given below to obtain an orthonormal basisβ for span(S) and compute the Fourier coefficients of the given vector x relative to $\beta. V = P_2(R)$ with inner product $\langle f(x), g(x)\rangle = \int_0^1 f(t)g(t)dt;\ S = \{1, x, x^2\}$ and $h(x) = 1 + x$.
18. Find the minimal solution of the following system of linear equations

$$x + 2y - z = 1$$
$$2x + 3y + z = 2$$
$$4x + 7y - z = 4$$

19. Find the minimal solution of the following system of linear equation
    $x + 2y + z = 4$

$$x - y + 2z = -11$$

$$x + 5y = 19$$

20. Let $V = P_2(R)$ with the inner product $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(t)g(t)dt$. consider the subspace $P_2(R)$ with standard ordered basis B. use the Gram Schmidt process to replace B by an orthogonal basis $\{v_1, v_2, v_3\}$ for $P_2(R)$ and use those orthogonal basis to obtain an orthonormal basis for $P_2(R)$.
21. For each of the sets of data that follow, use the lease square approximation to find the best fits with (i) linear function and (ii) a quadratic function. Compute the error E in both cases.
(a) $\{(1,2), (2,3), (3,5), (4,7)\}$.
(b) $\{(1,2), (3,4), (5,7), (7,9), (9,12)\}$
(c) $\{(-2,4), (-1,3), (0,1), (1,-1), (2,-3)\}$

ANSWERS

(a) Linear function $y = 1.7x, E = 0$
(b) Linear function $y = \frac{5}{4}x + \frac{11}{20}, E = \frac{3}{10}$ quadratic function $y = \frac{1}{56}x^2 + \frac{15}{14}x + \frac{239}{280}$, $E = \frac{8}{35}$
(c) Linear function $y = \frac{-9}{5}x + \frac{4}{5}, E = \frac{2}{5}$Quadratic function $y = \frac{-1}{7}x^2 - \frac{9}{5}x + \frac{38}{35}, E = \frac{4}{35}$

UNIT-IV PARTIAL DIFFERENTIAL EQUATIONS

PART A

1. Form the partial differential equation by eliminating the constants $a$and $b$from $z = (x^2 + a^2)(y^2 + b^2)$.[$AU\ Apr/May\ 2010$]
2. Form the partial differential equation by eliminating the constants $a$and $b$from $z = (x^2 + a)(y^2 + b)$.[$AU\ Apr/May\ 2011$]
3. Eliminate the arbitrary function $'f'$ from $z = f\left(\frac{y}{x}\right)$and form the PDE. [$AU\ Nov/Dec\ 2012$]
4. Form the partial differential equation by eliminating the arbitrary function from $z^2 - xy = f\left(\frac{x}{z}\right)$. [$AU\ Nov/Dec\ 2010,\ May/June\ 2012$]
5. Find the partial differential equation of all planes cutting equal intercepts from the x and y axes.[$AU\ Nov/Dec\ 2009$]
5. Find the PDE of the family of spheres having their centers on the z-axis.[$AU\ Nov/Dec\ 2011$]
6. Solve the partial differential equation $pq = x$.[$AU\ Apr/May\ 2010$]
7. Solve the equation $(D - D')^3 z = 0$.[$AU\ Nov/Dec\ 2011, Apr/May\ 2011$]
8. Solve the equation $(D^3 - 2D^2D')z = 0$.[$AU\ Nov/Dec\ 2009$]
9. Solve $\left(D^2 - 7DD' + 6D'^2\right)z = 0$.[$AU\ May/June\ 2012$]
10. Find the particular integral of $\left(D^2 - 2DD' + D'^2\right)z = e^{x-y}$. [$AU\ Nov/Dec\ 2010$]
11. Solve $(D - 1)(D - D' + 1)z = 0$. [$AU\ Nov/Dec\ 2012$]

PART B

❖ 1. PDE of Lagrange's Equation

1. Solve the partial differential equation $(mz - ny)p + (nx - lz)q = ly - mx$. [$\boldsymbol{AU\ Apr/May\ 2011}$]
2. Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.
3. Solve $y^2p - yxq = x(z-2y)$
4. Solve $\frac{y^2z}{x}p + xzq = y^2$
5. Solve $px^2 + qy^2 = z^2$
[$\boldsymbol{AU\ Nov/Dec\ 2010, May/June\ 2012}$]
6. Solve the partial differential equation $x(y-z)p + y(z-x)q = z(x-y)$. [$\boldsymbol{AU\ Nov/Dec\ 2011}$]
7. **Solve** $\boldsymbol{(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx}$
8. **Find the general solution of** $\boldsymbol{x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)}$
9. **Find the general solution** $(y-z)p - (2x+y)q = (2x+z)$
10. **Find the general solution** $x^2p + y^2q = z(x+y)$
11. **Find the general solution** $\boldsymbol{(x+2z)p + (2xz-y)q = x^2+y}$
12. Solve the partial differential equation $(y - xz)p + (yz - x)q = (x+y)(x-y)$. [$\boldsymbol{AU\ Nov/Dec\ 2009}$]
13. Solve the partial differential equation $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$.[$\boldsymbol{AU\ Apr/May\ 2010}$]
14. Solve the partial differential equation $(x-2z)p + (2z-y)q = y - x$. [$\boldsymbol{AU\ Nov/Dec\ 2012}$]
15. **Solve the partial differential equation** $\boldsymbol{x(y^2-z^2)p + y(z^2-x^2)q - z(x^2-y^2) = 0}$

❖ 2.Formation of PDE and Standard Types of PDE

16. Find the partial differential equation of all planes which are at a constant distance 'a'from the origin.[$\boldsymbol{AU\ Apr/May\ 2010}$]
17. Form the PDE by eliminating the arbitrary function $\varphi$from $\varphi(x^2+y^2+z^2,\ ax+by+cz) = 0$.
18. Form the PDE by eliminating the arbitrary function $\varphi$from $\varphi(x+y+z, x^2+y^2-z^2) = 0$.
19. Form the PDE by eliminating the arbitrary function φ from $\varphi(xyz, x^2+y^2+z^2) = 0$
20. Form the PDE by eliminating the arbitrary function φ from $\varphi\left(z^2 - xy,\ \frac{x}{z}\right) = 0$
21. Form the PDE by eliminating the arbitrary function φ from $f(xy+z^2, x+y+z) = 0$
[$\boldsymbol{AU\ Nov/Dec\ 2010, May/June\ 2012}$]
22. Form the partial differential equation by eliminating arbitrary functions $f$and $\varphi$from $z = f(x+ct) + \varphi(x-ct)$.[$\boldsymbol{AU\ Apr/May\ 2011}$]
23. Solve $= px + qy + p^2q^2$. [$\boldsymbol{AU\ Nov/Dec\ 2009}$]
24. Find the singular integral of $z = px + qy + \sqrt{1+p^2+q^2}$. [$\boldsymbol{AU\ Nov/Dec\ 2011}$]
25. Find the singular integral of $z = px + qy + p^2 + pq + q^2$. [$\boldsymbol{AU\ Nov/Dec\ 2012}$]
26. **Solve** $\boldsymbol{z = px + qy + \left(\frac{q}{p} - p\right)}$
27. **Solve** $\boldsymbol{z = px + qy + pq}$
28. Solve $p(1+q) = qz$. [$\boldsymbol{AU\ Apr/May\ 2010}$]
29. Solve $p^2 + q^2 = x^2 + y^2$. [$\boldsymbol{AU\ Apr/May\ 2010}$]
30. **Solve** $\boldsymbol{z^2(p^2+q^2) = x^2+y^2}$.
31. **Solve** $\boldsymbol{p + q = \sin x + \sin y}$

32. **Solve $z^2 = p^2 + q^2 + 1$**

33. **Solve $\sqrt{p} + \sqrt{q} = \sqrt{x}$**

❖ 3.Homogeneous Linear Partial Differential Equation

34. Solve $\left(D^2 + 2DD' + D'^2\right)z = \sinh(x+y) + e^{x+2y}$.[$AU\ Nov/Dec\ 2009$]

35. Solve$\left(D^3 + D^2D' - 4DD'^2 - 4D'^3\right)z = \cos(x+2y)$.[$AU\ Nov/Dec\ 2010, May/June\ 2012$]

36. Solve $\left(D^2 - D'^2\right)z = e^{x-y}\sin(2x+3y)$.[$AU\ Apr/May\ 2011$]

37. Solve $\left(D^3 - 2D^2D'\right)z = 2e^{2x} + 3x^2y$.[$AU\ Nov/Dec\ 2011$]

38. Solve $\left(D^2 + 3DD' - 4D'^2\right)z = \cos(2x+y) + xy$.[$AU\ Nov/Dec\ 2012$]

39. **Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \cos(x+2y) + x$.**

40. **Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x+2y) + e^{3x+y}$.**

❖ 4.Non Homogeneous Linear Partial Differential Equation

41. Solve $\left(D^2 - D'^2 - 3D + 3D'\right)z = xy + 7$.[$AU\ Nov/Dec\ 2009$]

42. Solve $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = \sin(x+2y)$.[$AU\ Apr/May\ 2010$]

43. Solve $\left(2D^2 - DD' - D'^2 + 6D + 3D'\right)z = xe^y$.[$AU\ Nov/Dec\ 2010, May/June\ 2012$]

44. Solve $\left(D^2 - 3DD' + 2D'^2 + 2D - 2D'\right)z = x + y + \sin(2x+y)$.[$AU\ Apr/May\ 2011$]

45. Solve $\left(D^2 - 2DD' + D'^2 - 3D + 3D' + 2\right)z = e^{2x-y}$.[$AU\ Nov/Dec\ 2011$]

46. Solve $\left(D^2 - DD' - 2D\right)z = e^{2x+y} + 4$.[$AU\ Nov/Dec\ 2012$]

UNIT V FOURIER SERIES SOLUTIONS OF PDE

PART A

1. Write the conditions for $f(x)$ to satisfy for the existence of a Fourier series.
2. State whether $y = \tan x$ can be expanded as a Fourier series, if so how? If not why?
3. Find the value of a_0 in the Fourier expansion of $f(x) = e^x$ in $(0, 2\pi)$
4. Find the value of a_n in the Fourier expansion of $f(x) = e^{-x}$ in $(-\pi, \pi)$
5. Find the value of a_2 in the Fourier expansion of $f(x) = 2x$ in $(0,4)$
6. Find the coefficient b_n of the Fourier series for the function $f(x) = x\sin x$ in $(-2,2)$
7. If $x^2 = \frac{\pi^2}{3} - 4\sum_1^\infty \frac{(-1)^{n-1}}{n^2} cosnx$in $-\pi < x < \pi$. Deduce that $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$.
8. Find the constant term in the expression cos^2x as a Fourier series in $(-\pi, \pi)$
9. In the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$ what does a^2 stands for?
10. Write all possible solutions of one dimensional wave equation.
11. State the assumption made in the derivation of wave equation

12. In the one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 stands for?
13. Write down the possible solutions of one dimensional heat flow equation
14. Write down the solution of Laplace equation which is periodic in y
15. What is the basic difference between the solution of one dimensional wave equation and one dimensional heat flow equation with respect to time
16. Write down steady state heat flow equation in one dimension and its solution.
17. Define steady state solution on heat flow.
18. Write down the possible solutions of two dimensional steady state heat flow or Laplace equation.
19. An insulated rod of length l cm has its ends A and B krpt at $0^\circ c$and $80^\circ c$. Find the steady state condition of the rod.
20. A plate is bounded by the lines $x = 0, y = 0, x = l, and\ \ y = l$. The edge coinciding with x axis is kept at $100^\circ c$. The edge coinciding with y axis is kept at $50^\circ c$. The other two edges are kept at $0^\circ c$. Write down the boundary conditions.
21. The ends A and B of rod of length 20 cm have their temperatures kept at $10^\circ c$ and $50^\circ c$ respectively. Find the steady state temperature.
22. A rod 30 cm long has its ends A and B kept at $20^\circ c\ and\ 80^\circ c$ respectively until steady state conditions prevail. Find the steady state temperature.
23. Write down the two dimensional heat equations both in transient and steady state.

PART B

1. Find the Fourier series for $f(x) = (\pi - x)^2$ in the interval $(0, 2\pi)$ and deduce sum of the series as $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots$..
2. Obtain the Fourier series of $f(x)$ of period $2l$ and defined as follows $f(x) = \begin{cases} l - x & 0 < x < l \\ 0 & l < x < 2l \end{cases}$ and hence deduce the following (i) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots \infty = \frac{\pi}{4}$ (ii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots .. = \frac{\pi^2}{8}$
3. Find Cosine series for $f(x) = x$ in $(0, \pi)$
4. Obtain sine series for $f(x) = \begin{cases} x, & 0 < x < l/2 \\ l - x, & l/2 < x < l \end{cases}$
5. Find the Fourier series of $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$. Hence deduce $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots \infty = \frac{\pi}{4}$
6. Show that when $0 < x < \pi$, $\pi - x = \frac{\pi}{2} + \frac{\sin 2x}{1} + \frac{sin4x}{2} + \frac{\sin 6x}{3} + \cdots$...
7. Develop a sine series of the function $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$
8. Find the Fourier series of $f(x) = x(2 - x)$ in $0 < x < 2$. deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots .. \infty$
9. Find half range cosine series for $f(x) = \pi - x^2$in $(0, \pi)$.

10. Find the Fourier series expansion of $x\cos x$ in $-\pi < x < \pi$
11. Find the Fourier series to represent $f(x) = |\cos x|$ in $-\pi < x < \pi$
12. Find the Fourier series to represent $f(x) = |sin\, x|$ in $-\pi < x < \pi$
13. Obtain the Fourier series to represent the function

$f(x) = |x|, -\pi < x < \pi$ and deduce $\frac{1}{1^2} + \frac{1}{3^2} + \cdots \ldots \ldots \ldots \ldots$

14. Find the Fourier series of the function $f(x) = \begin{cases} 0, -\pi \le x \le 0 \\ sinx, 0 \le x \le \pi \end{cases}$ and prove

    that (i) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots \ldots = \frac{1}{2}$ (ii) $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \cdots \ldots = \frac{\pi-2}{4}$

15. Find the Fourier series expansion of $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \end{cases}$. Also deduce

    the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots \ldots \ldots .. \infty = \frac{\pi^2}{8}$

16. Show that $0 \le x \le \pi, \pi x - x^2 = \frac{\pi^2}{6} - 4\left[\frac{cos2x}{2^2} + \frac{cos4x}{4^2} + \frac{cos6x}{6^2} + \cdots \ldots \ldots ..\right]$

    Hence deduce the value of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots \ldots ..$

17. Obtain the Fourier cosines series expansion of $xsinx\ in\ (0, \pi)$ and hence

    find the value of $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} - \frac{2}{7.9} + \cdots \ldots \ldots$

18. By finding the sine series for $y = x$ in $0 < x < l$ show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

19. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $kx(l-x)$, $0 < x < l$, determine the displacement .
20. If a string of length l is initially at rest in its equilibrium position and each of its points given a $y(x,0) = \begin{cases} cx & 0 < x < \frac{l}{2} \\ c(l-x) & \frac{l}{2} < x < l \end{cases}$. Find the displacement.
21. A string of length $2l$ is fastened at both ends. The midpoint of the string is taken to a height b and then released from rest in the position. Find the displacement of the string.
22. The points of trisection of a string of length l are pulled aside through a distance h on opposite sides of the position of equilibrium and the string is released from rest. Find an expression for the displacement of the string at any subsequent time. Also show that the midpoint of the string always remains at rest.
23. A square plate is bounded by the lines $x = 0,\ x = a, y = 0,\ y = a$ are kept at temperature $0^{\circ} c$. The side $x = a$ is kept at temperature given by $u(a, y) = 100, 0 < y < a$.
24. The ends A and B of a rod 30cm long have their temperature at $20^{\circ} c$ and at $80^{\circ} c$ until steady state conditions prevail. The temperature at each end is then suddenly

reduced to to$0^{\circ}c$ and kept so. Find the temperature distribution in a regular function $u(x,t)$ taking $x = 0$ at A.

25. A rectangular plate with insulated surface is 20 cm wide and so long compared to its width that it may be infinite in length without introducing an appreciable error. If the temperature of the short edge $x = 0$ is given by $u = \begin{cases} 10\,y & 0 < y < 10 \\ 10(20-y) & 10 < y < 20 \end{cases}$ and the two long edges as well as the other short edge are kept at$0^{\circ}c$. Find the steady state temperature distribution in the plate.
26. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 sin^3\left(\frac{\pi x}{l}\right)$. It is released from the rest from this position. Find the expression for the displacement at any time$t$.
27. A uniform string is stretched and fastened to two points $l$apart.motion is started by displacing the string into the form of the curve $y = kx(l-x)$ and then released from this position at time $t = 0$. Derive the expression for the displacement of any point of the string a distance x from one end at time t.
28. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially displaced to the form $2\,sin\left(\frac{3\pi x}{l}\right)cos\left(\frac{2\pi x}{l}\right)$ and then released from rest from this position. Find the displacement of the string any point distance from this position. Find the expression for the displacement at any time t.
29. A tightly stretched string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x,t)$.
30. A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be infinite in length without introducing an appreciable error. If the temperature of the short edge $y = 0$ is given by $u = \begin{cases} 20\,x & 0 \le x \le 5 \\ 20(10-x) & 5 \le x \le 10 \end{cases}$ and the two long edges as well as the other short edge are kept at $0^{\circ}c$. Find the steady sate temperature distribution in the plate.
31. A square plate is bounded by the lines $x = 0, y = 0, x = 20$and $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,20) = x(20-x),\ 0 < x < 20$ while the other three edges are kept at $0^{\circ}c$. Find the steady state temperature distribution in the plate.
32. A long rectangular plate with insulated surface is $l\ cm$ wide. If the temperature along one short edge $y = 0$ is $u(x,0) = k(lx - x^2)$ degrees, for $0 < x < l$ while the other two long edges $x = 0$ and $x = l$ as well as the other short edge are kept at $0^{\circ}c$, find the steady state temperature function $u(x,y)$.